

Use of Mathematics in Music Composition

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Abstract

This extended essay is about the relationship of music and mathematics, whether mathematics is used in music composition. If so how and whether use of mathematics is necessary to compose music. Today the interest in relationship between music mathematics is rose which started from the ancient. Many scholars analyze mathematics in music and further more argue whether the musicians used mathematics on purpose or not.

The scope of this essay compares and contrasts music and mathematics by defining each subject since ancient Greek to modern period and discussing purpose of each subject. Then it explores what mathematics is used in music by introducing well known musicians such as J.S. Bach and W.A. Mozart whose work can be analyzed with mathematics and how the musicians used such series and ratios. Finally it argues whether mathematics is necessary for composing.

It is clear that music and mathematics are related each other through the concept of numbers presented by the proportions and ratios. Some musicians attempted to apply mathematics in their works. But, then is it necessary to use mathematics in music composition? My answer to this question was that it is not necessary because the musicians used mathematics more consciously than on purpose and proportions and ratios rather occurred naturally.

208 words

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Use of Mathematics in Music Composition

Probably I have heard music even before I was born because my father is a singer I probably have gone to his concert. I learned '1+1=2' before entering elementary school. For 18 years or more of my life I was surrounded by music and mathematics. I have studied music, music theory and mathematics for a long time and met these two subjects pretty much every day. So I was familiar with two subjects but I just thought music and mathematics as two really separated fields. I haven't really thought about the relationship between two subjects like how are they related before I chose 'Use of Mathematics in Music Composition' as my topic for extended essay. One day I have heard about Mozart using golden ratio. It was really interested and I started research in depth through writing this essay.

Music has changed meaning as time passes. Now we think music is sound which can be played by a voice or an instrument in general. Originally, the word music comes from the Greek *mousike* derived from *muse* who is Greek goddess of the art and science. In ancient Greek, *mousike* meant "any arts or sciences governed by the Muses" (Knowledgerush). In the ancient, music was close to conceptualized form. In modern definition of music is "the science or art of ordering tones or sounds in succession, in combination and in temporal relationships to produce a composition having unity and continuity" which seems focus on harmony as a result of proportions and another typical definition is "vocal, instrumental, or mechanical sounds having rhythm, melody, or harmony" (Merriam-Webster).

In Middle Ages of Europe they had thoroughly combined thought of science and mathematics to conceptualize music. It is expansion of Pythagoras ratio from ancient time. Although today mathematicians can prove the relationship between music and mathematics using more complicated equation there was an idea about conceptual music related to simple

mathematical proportion even in the past. Musica, who are “the sounds people accept as music according to historical era”, was part of the mathematical quadrivium which includes arithmetics, geometry, astronomy and musica. The concept of musica was split into three groups-musica universalis or mundana, musica humana and musica instrumentalis.

Musica universalis which is also known as Music of the Spheres or Musica mundane is “not audible, but simply a mathematical concept” (Crystalinks). So people believed that “God had created it in measure, number and weight” and perceived “the proportions of the spheres of the planets and stars as a form of music” (Knowledgerush). This says people considered music related to mathematics in the past. Musica humana is “microcosmic harmony of the body and soul” (Christensen 3). Music was focused soul with harmony which is emotion and close to today’s meaning of music. Yet microcosmic harmonies related to number that can basic relationship with mathematics. Musica instrumentalis refers to “the manifestation of those same mathematical proportions in sound” (Knowledgerush), so it is actual music sung or played by instruments that we generally conceptualize music recently.

In our society we are surrounded by many mathematical factors because mathematics is basic subject for many other subjects from natural sciences, engineering, and computer sciences even to social sciences such as economy, business. There are more than these fields that uses mathematics. Mathematics is “a study of patterns and relationships; a science and a way of thinking; an art, characterized by order and internal consistency; a language, using carefully defined terms and symbols; and a tool” (Learning Point). Mathematics has various roles in our lives and people use practical mathematics as a tool in various ways. Mathematics is much more logical and practical subject compared to music which works more with emotions and souls.

How are two subjects so different from each other? Music does not have specific form to fit into certain place or time which means it is interchangeable. It can be clear to listen to and also it can be really hard to listen to or understand like abstract art. On the other hand mathematics has certain format and axiom and these do not change through the place or time. Somehow music does not have absolute necessity. Perhaps society could run and won't have any big differences without music whereas mathematics has very important roles. Music is related to emotions and understanding can change depends on people and time, whereas mathematics focuses on logic and truth so people can understand same materials and truth that was passed on. As we see music and mathematics seem apart from each other; they come from so different roots. Their focus and borders differ two subject areas; between beauty and truth, between change or alteration and eternity, and between freedom and limitation.

What music and mathematics have in common is that we can share these using common senses; we see the mathematics and we hear the music. Even though they act differently they have something in common. "Plato had already written that the eye is affected by optical impressions in quite the same way as the ear is affected by sounds" (Escot 8). As Plato said previously the effect of sight and effect of acoustic are same way; if you can see through hearing and if you can hear through seeing then you can hear through seeing and also you can see through hearing. They relate and help each other. This relationship shows closely related music and mathematics.

Both music and mathematics are language as themselves; they can express what they possess by letting us hear and showing. Dr. David Moursund says that "Mathematics is an interdisciplinary language and tool. Like reading and writing, math is an important component of learning and 'doing' (using one's knowledge) in each academic discipline. Mathematics is such a

useful language and tool that it is considered one of the 'basics' in our formal educational system." Also in *Emblems of Mind*, the author says "the language of mathematics will provide useful metaphors for understanding music." One language can be translated to another language therefore music and mathematics are also convertible. Translation is not necessarily exactly same each other but the important point is that they share same ideas. As music used mathematical fact, they share their ideas.

Recently, relationship between music and mathematics rose popularly among the people. Music and mathematics seem like really different area because mathematics is really logical whereas music has more freedom without boundaries. However, on the other hand, mathematics is really aesthetic by using perfect dynamic symmetry and music shares this basic idea of dynamics and symmetries. In *Emblems of Mind*, Rothstein says, we try to find similarities within differences and through this process we find common fact, share fact, and relations.

Relationship between music and mathematics revealed from way back to Pythagoras. He was the first person who tried to make the connection between these two different subjects. Pythagoras explored his idea about harmony named as Pythagoras tuning which influenced "both the ancient and the modern world" (Barbour 1). Tuning is the ratio between notes of sounds and basically the Pythagoras tuning is about certain proportions and ratios between two sounds. Pythagoras tuning is based upon the octaves and fifth of the harmonic series between two intervals. Pythagoras uses "ratios of 2:1 for the octave and 3:2 for the fifth" to tune "all the notes of the diatonic scale in a succession of fifths and octaves" or "all the notes of the chromatic scales" (Barbour 1). Mathematical principle underlies the Pythagoras tuning.

Relationship between music and mathematics can be said relationship between sound and mathematics. It means that sound is very important as the basic element of music. As mentioned

above in the Pythagoras tuning sound is closely related to mathematics. Music is order of sound in order to make beautiful harmony. To compose harmonies the proportions and the ratios are important. The proportions and the ratios are all about numbers. In addition, sound is from rhythm which is various kinds of beat through time. The concept of beat is also closely related to numbers.

On the side, there is a theory which explains right brain is involved in melody and left brain is involved in rhythm in the brain studies. It then can explain that there is certain part of brain work for each component of music which can be focused on mathematics such as rhythm and focused on musical element such as melody. This theory helps to indicate certain musicians who show their use of specific part of brain. One hand, some musicians who have worked not only music but also art, literature composes romantic and who use both left and right brain. On the other hand, some musicians, for example, Bach, who only worked on music, used left brain mostly which means he composed mathematical and exquisite music.

Johann Sebastian Bach was a German composer and organist in Baroque period which focused on theory and composing was controlled which means music was made on purpose. Mathematicians are interested in Bach's music because they could find some mathematical facts in his music. Indeed, Bach sometimes used mathematical facts such as equal temperament and Fibonacci number.

Bach is the first composer who used equal temperament into the composition. Equal temperament is temperament dividing an octave with 12 halftones like piano. It does not completely agree with pure temperament which is composed with 8 tone scale that makes people feel comfortable and natural sound but it leads orchestra with instruments which has tune that can not be changed. Kottick says that Bach "wanted to demonstrate the potentialities of unequal

temperament,” showing through the version known as *Wohltemperatur*, or well-temperament (154). I think this process led Bach to use equal temperament. For a long time, generations of pianists believed that “*Wohltemperatur* referred to equal temperament” and also believed that Bach was expert on using equal temperament (Kottick 154). However, Kottick has little doubt that Bach was aware of the theory of equal temperament because by that time the theory had been too old around for at least two thousand years (154).

Bach also often used Fibonacci number in his music. Fibonacci sequence is growing by adding two previous numbers starting from two 1s. Therefore Fibonacci sequence is 1, 1, 2, 3, 4, 8, 13, 21, 34, 55, 89... and so on. As I mentioned before Bach used left brain mostly when he composed and that means he focused on rhythm and scales and Fibonacci sequence helped to develop scales by increasing number of measures notes. Trudi H Garland points out that the 5-tone scale represented by the black notes on the piano and the 8-tone scale represented by the white notes on the piano and the 13-tone scale which is a complete octave in semitones including the two notes an octave apart. 8-tone scale counts the same note C twice which is called an octave. Oct in octave means eight. The octave completes the cycle by repeating the starting note at the end and it sounds more pleasing to the ears. It shows that Fibonacci number makes more pleasing sound and harmony in music.

B. Bartok also based most structure on Fibonacci sequence and Golden Ratio. The significance of Fibonacci sequence is that overall music goes up from start with pianissimo to climax with fortissimo and goes down from climax to end with pianissimo again.

Wolfgang Amadeus Mozart was a German composer in Classical period which stands strongly against Baroque style. He used Golden Ratio which makes his music interesting. Golden Ratio which is also known as Golden Section is a very famous not only in music but also in other

subjects such as visual art as well. For example when music has a hundred measures then climax is probably at 62nd measure and this is ideal harmony to hear what musician and also mathematician consider.

Perhaps Mozart used mathematics when he was composing but we cannot know whether he used on purpose or not. We can only guess, argue and discuss. One article issued by *American Scientist* argues that Mozart did not use Golden Section for his composing and it supports my argument that use of mathematics is not necessary all the time for the music composition. Because music has enough elements and principals itself, music does not have to depend on use of mathematics. Although there is a fact never change that music and mathematics are related each other and sometimes they affect each other, the necessity of using mathematics in music composition is not explained by those ideas.

Chamber Music Today leads attention to Sofia Gubaidulina's compositions which has Fibonacci series, Golden Ratio and other mathematical constructs. However it also says that "it's unclear whether Sofia actually uses the words 'numerology' or 'gematria' in discussing her own compositional methods," because she believes spiritual practices in the backgrounds. We can analyze ratio in her composition but it does not mean she used mathematical method necessarily because "even if you don't measure it, you feel it." Music is more about you feel it rather than you measure it while creating and listening, and the article says there is a "naturalness" in her rhythmic structures. Because of the proportion and statistical distributions of the durations of the notes music seem like having certain ratio; however, it is close to natural phenomenon. Conclusion of this article is that "If a composer sticks scrupulously to a mathematical construct in devising rhythm and orchestration, the result may be too complex to be playable."

Through the research I found that music and mathematics certainly have relationships mainly related to numbers and sound. However theoretically and also ideally music can be composed without mathematical help. Mathematical formulas can be used to make interesting patterns and balance, but harmony itself can be made from musician's spirit. Mozart, Bach and other musicians who made amused music are probably believed and relied on more their emotions, souls and spirits than numbers and mathematical formulas. Also the musicians would have instinct ability to balance and to make harmony when they created music. So then people who are listening to music afterwards can be attracted and analyze mathematics inside of music. Music and mathematics were and are developed through communicating each other. But if one subject relies on other subject too much, then it would be hard to end with pure products.

2353 words

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